BER decreases with DIVERSITY.

How to characterize Diversity of a system

'Diversity order'

Diversity order — d.

$$d = -\lim_{SNR \to \infty} \frac{\log Pe(SNR)}{\log SNR}$$

where $Pe(SNR)$ denotes Prob of Error as a function of SNR.

Example:

Consider a wireless system with $L = 1$ antenna.

Diversity order?

high SNR

$$P_e(SNR) \approx \frac{1}{2 \cdot SNR}$$

$$d = -\lim_{SNR \to \infty} \frac{\log P_e(SNR)}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{\log \frac{1}{2SNR}}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{-\log SNR - \log 2}{\log SNR}$$

$$= \lim_{SNR \to \infty} 1 + \frac{\log 2}{\log SNR}$$

$\frac{\log 2}{\log SNR} \to 0$

$$d = \lim_{SNR \to \infty} 1 + \frac{\log 2}{\log SNR}$$

$\log SNR \to 0$

$$\boxed{d = 1}$$

diversity order of system for $L = 1$ is $d = 1$.

Consider a system with L antennas.

$$P_e(SNR) = {}^{2L-1}C_L \cdot \left(\frac{1}{2\,SNR}\right)^L$$

$$= {}^{2L-1}C_L \cdot \frac{1}{2^L} \cdot \left(\frac{1}{SNR}\right)^L$$

$$d = -\lim_{SNR \to \infty} \frac{\log {}^{2L-1}C_L \frac{1}{2^L} \cdot \frac{1}{SNR^L}}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{-L \log SNR + \log {}^{2L-1}C_L \frac{1}{2^L}}{\log SNR}$$

$$= \lim_{SNR \to \infty} L - \frac{\log {}^{2L-1}C_L \frac{1}{2^L}}{\log SNR} \quad (\log SNR \to 0)$$

$$= L$$

Diversity order of system with L antennas $= L$.

What is diversity order of wireline system?

$$P_e(SNR) = Q\left(\sqrt{SNR}\right)$$

$$\approx \frac{1}{2} e^{-\frac{1}{2} SNR}$$

$$d = -\lim_{SNR \to \infty} \frac{\log \frac{1}{2} e^{-\frac{1}{2} SNR}}{\log SNR}$$

$$= -\lim_{SNR \to \infty} \frac{\log \frac{1}{2} - \frac{1}{2} SNR}{\log SNR}$$

$$= \lim_{SNR \to \infty} \frac{\frac{1}{2} SNR}{\log SNR}$$

Using L'Hospital's rule for limits,

$$d = \lim_{SNR \to \infty} \frac{\frac{1}{2}}{\frac{1}{SNR}}$$

$$= \lim_{SNR \to \infty} \frac{SNR}{2} = \infty$$

diversity order of AWGN or wireline channel = ∞

AWGN channel is a combination of ∞ independently faded links.